PHHQP15: Book of Abstract.

Part I: Talks.
Part II: Posters.
Some subtle features of complex PT-symmetric quantum mechanics in one dimension

Zafar Ahmed

Abstract

We revisit quantum mechanics of one-dimensional complex PT-symmetric potentials to raise and address the following questions. Do the real discrete eigenvalues occur as doublets; what are these doublets? When PT-symmetry breaks down spontaneously, how the eigenstates of complex-conjugate energy eigenvalues act under PT? Can real discrete eigenvalue curves cross when a potential-parameter is varied slowly, will the degeneracy occur? For a scattering potential with real discrete spectrum are there two parametric domains of spontaneous breaking of PT-symmetry: one with regard to bound states and other for scattering states? Can spectral singularity occur in the domain of unbroken PT-symmetry? If a spectral singularity occurs at $E = E^*$ for a complex scattering potential, what would it mean for its PT-transformed counterpart?

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PT symmetric classical and quantum cosmology

Alexander A. Andrianov∗,
joint with
Oleg O. Novikov, Chen Lan

Abstract

According to modern observational data the phantom equation of state for dark energy is not excluded. This scenario creates problems with stability of universe evolution. An alternative to phantom models is given by models of scalar matter with non-Hermitian, PT-symmetric interaction[1]. In this talk we present the set of such models with several scalar fields which are exactly (analytically) solvable both in classical and in quantum case[2]. The latter case is investigated in the FRW minisuperspace approach. We use advantages of analytical solvability to disentangle the differences between phantom and PT symmetric dynamics and give a favor to the latter one.

References


Metric operators, generalized hermiticity and lattices of Hilbert spaces

Jean-Pierre Antoine∗
joint with
Camillo Trapani†

Abstract

Motivated by the recent developments of pseudo-Hermitian quantum mechanics, we analyze the structure generated by metric operators, bounded or unbounded, in a Hilbert space. We introduce the notions of similarity and quasi-similarity between operators, and we explore to what extent they preserve spectral properties. Then we reformulate the notion of quasi-Hermitian and pseudo-Hermitian operators in the preceding formalism.

Next we consider canonical lattices of Hilbert spaces generated, first by a single metric operator, then by a family of metric operators. Since such lattices constitute the simplest case of a partial inner product space (PIP-space), we can exploit the technique of PIP-space operators. In particular we investigate the quasi-similarity of PIP-space operators, with particular emphasis on symmetric PIP-space operators, which are candidates for Hamiltonians of physical systems.

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Nonlinear Schrödinger dimer with gain and loss:
integrability and $\mathcal{PT}$-symmetry restoration

Igor Barashenkov∗
joint with
Dmitry Pelinovsky† and Philippe Dubard∗

Abstract

A $\mathcal{PT}$-symmetric nonlinear Schrödinger dimer is a two-site discrete nonlinear Schrödinger equation with one site losing and the other one gaining energy at the same rate. A physically important example (occasionally referred to as the standard dimer) is given by [1, 2, 4]:

\[ i\dot{u} + v + |u|^2 u = i\gamma u, \quad i\dot{v} + u + |v|^2 v = -i\gamma v. \]

Another model with a wide range of applications is [3]

\[ i\dot{u} + v + (|u|^2 + 2|v|^2) u + v^2 u^* = i\gamma u, \]
\[ i\dot{v} + u + (|v|^2 + 2|u|^2) v + u^2 v^* = -i\gamma v. \]

We construct two four-parameter families of cubic $\mathcal{PT}$-symmetric dimers as gain-loss extensions of their conservative, Hamiltonian, counterparts. Our main result is that all these damped-driven discrete Schrödinger equations define completely integrable Hamiltonian systems. Furthermore, we identify dimers that exhibit the nonlinearity-induced $\mathcal{PT}$-symmetry restoration. When a dimer of this type is in its symmetry-broken phase, the exponential growth of small initial conditions is saturated by the nonlinear coupling which diverts increasingly large amounts of energy from the gaining to the losing site. As a result, the exponential growth is arrested and all trajectories remain trapped in a finite part of the phase space regardless of the value of the gain-loss coefficient.

References

[2] I V Barashenkov, G S Jackson, S Flach, Blow-up regimes in the $\mathcal{PT}$-symmetric coupler and the actively coupled dimer. PRA 88 (2013) 053817
[4] I V Barashenkov, Hamiltonian formulation of the standard $\mathcal{PT}$-symmetric nonlinear Schrödinger dimer. PRA 90 (2014) 045802

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Hamiltonians expressed in terms of bosonic operators, and their spectra

Natália Bebiano*
joint with
João da Providência†

Abstract

We investigate spectral aspects of non self-adjoint bosonic operators. We show that the so-called equation of motion method, which is well known from the treatment of self-adjoint bosonic operators, is also useful to obtain the explicit form of the eigenvectors and eigenvalues of non self-adjoint bosonic Hamiltonians with real spectrum. We also demonstrate that these operators can be diagonalized when they are expressed in terms of quasi-bosons, which do not behave as true bosons under the adjoint transformation, but still obey the Weil-Heisenberg algebra.

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Riesz-like bases in rigged Hilbert spaces and Pseudo-Hermitian Quantum Mechanics

Giorgia Bellomonte

Abstract

We propose a construction of the physical Hilbert space for quantum systems defined by unbounded metric operators. This construction applies for Hamiltonian operators $H$ with a discrete real spectrum and is carried out by means of what we called strict Riesz-like bases for rigged Hilbert spaces. Differently from Riesz bases, a Schauder basis $\{\xi_n\}$ for $\mathcal{D}[t]$ is called a strict Riesz-like basis for $\mathcal{D}[t]$ if there exists a continuous operator $T : \mathcal{D}[t] \to \mathcal{H}[\|\cdot\|]$ that makes of $\{T\xi_n\}$ an orthonormal basis for $\mathcal{H}$ and has continuous inverse $T^{-1} : \mathcal{H}[\|\cdot\|] \to \mathcal{D}[t]$ (in particular, $T^{-1} \in B(\mathcal{H})$). We also prove that every $\omega$-independent, complete (total) Bessel sequence is a (strict) Riesz-like basis in a convenient triplet of Hilbert spaces. Moreover, some of the simplest operators we can define by them and their dual bases are studied.

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Nonlinear eigenvalue problems and $\mathcal{PT}$-symmetric quantum mechanics

Carl M. Bender*

Abstract

We discuss new kinds of nonlinear eigenvalue problems, which are associated with instabilities, separatrix behavior, and hyperasymptotics. We consider first the differential equation $y'(x) = \cos[\pi xy(x)]$, which arises in a number of physical contexts. We show that the initial condition $y(0)$ falls into discrete classes: $a_{n-1} < y(0) < a_n$ ($n = 1, 2, 3, \ldots$). If $y(0)$ is in the $n$th class, $y(x)$ exhibits $n$ oscillations. The boundaries $a_n$ of these classes are strongly analogous to quantum-mechanical eigenvalues and calculating the large-$n$ behavior of $a_n$ is analogous to performing a semiclassical (WKB) approximation in quantum mechanics. For large $n$, $a_n$ is asymptotic to $A\sqrt{n}$, where $A = 2^{5/6}$ [1]. Surprisingly, the constant $A$ is numerically close to the lower bound on the power-series constant $P$, which plays a fundamental role in the theory of complex variables and which is associated with the asymptotic behavior of zeros of partial sums of Taylor series [1].

The first two Painlevé transcendents $P_1$ and $P_2$ have eigenvalue structures just like that of $y'(x) = \cos[\pi xy(x)]$. As $n \to \infty$, the $n$th eigenvalue for $P_1$ grows like $Bn^{2/5}$ and the $n$th eigenvalue for $P_2$ grows like $Cn^{2/3}$. We calculate the constants $B$ and $C$ analytically by reducing the Painlevé transcendents to linear eigenvalue problems in $\mathcal{PT}$-symmetric quantum theory [2].

References


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Abstract

Abstract: There have been some confusion / controversy recently in the literature regarding the status of combined systems (entanglement, etc.) in PT-symmetric quantum mechanics [Lee et al. Phys. Rev. Lett. 112, 130404 (2014); Chen et al. Phys. Rev. A90, 054301 (2014)]. The purpose of this talk is to clarify the situation by exploiting the biorthogonal formulation of PT-symmetric quantum mechanics.

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Discrete and continuous frame expansions in Hilbert spaces and coherent states

Ole Christensen

Abstract

During the past 20 years the mathematical community has experienced an increasing interest in series expansions in terms of redundant systems, the so-called frames. We will give an overview of the general frame theory in Hilbert spaces, as well as the concrete manifestations of the theory in the setting of time-frequency analysis and wavelet analysis. Both cases fall within the general setting of coherent states.

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Mesuring a reservoir spectral density via one-dimensional photon scattering

Francesco Ciccarello*

Abstract

The spectral density (SD) function has a central role in the study of open quantum systems (OQSs). We show [1] a spectroscopic method for measuring the SD, requiring neither the OQS to be initially excited nor its time evolution tracked in time, which is not limited to the weak-coupling regime. This is achieved through one-dimensional photon scattering for a zero-temperature reservoir coupled to a two-level OQS via the rotating wave approximation. We find that the SD profile associated with the OQS own reservoir can be exactly mapped into a universal simple function of the photon’s reflectance and transmittance [1]. As such, it can be straightforwardly inferred from photon’s reflection and transmission spectra.

References


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Non-Hermitian noncommutative models in quantum optics and their superiorities

Sanjib Dey∗
joint with
Véronique Hussin†

Abstract

We find the dual nature of the nonlinear coherent states in noncommutative space [2]. The classical-like nature follows from the fact that the generalised uncertainty relation is saturated [1]. While, the nonclassical behaviour is analysed from the property of quadrature squeezing, which makes them more interesting from the quantum optical point of view [1]. We utilize the models, which are non-Hermitian by nature and lead to the existence of minimal length. We construct the coherent states, cat states and squeezed states in the noncommutative space and compute the entanglement in each case [2]. We demonstrate the superiorities of utilising the noncommutative structure by comparing the outcomes with the usual quantum mechanical systems [1, 2].

References


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Euclidean algebras in Quasi-Hermitian quantum systems

Andreas Fring∗

Abstract

We argue that Euclidean algebras constitute the natural framework for a large class of quasi-Hermitian systems, especially those related to optical systems. For many models based on these Lie algebras the Dyson map, metric with corresponding PT-symmetry breaking at the exceptional points can be constructed explicitly. We propose how these algebras can be employed to build a framework for quasi-exact solvability. Some examples considered reduce to the complex Mathieu Hamiltonian in a double scaling limit, which enables us to compute the exceptional points in the energy spectrum of the latter as a limiting process of the zeros for some algebraic equations. The coefficient functions in the quasi-exact eigenfunctions are univariate polynomials in the energy obeying a three-term recurrence relation. The latter property guarantees the existence of a linear functional such that the polynomials become orthogonal. The polynomials are shown to factorize for all levels above the quantization condition leading to vanishing norms rendering them to be weakly orthogonal. In two concrete examples we compute the explicit expressions for the Stieltjes measure.

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Pseudo Fermions: connection with exceptional points and $\mathcal{PT}$ quantum mechanics

Francesco Gargano∗
joint with
Fabio Bagarello †

Abstract
We discuss the role of pseudo-fermions in the analysis of some two dimensional models recently introduced in connection with non self-adjoint Hamiltonians ([1, 2, 3, 4]). Among other aspects, we discuss the link between the pseudo-fermions structure with the $\mathcal{PT}$-symmetric quantum mechanics and the appearance of exceptional points in connection with the validity of the extended anti-commutation rules which define the pseudo fermions ([5]).

References

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Bound states, scattering states and resonant states in $\mathcal{PT}$-symmetric open quantum systems

Savannah Garmon∗

joint with
Mariagiovanna Gianfreda†, Naomichi Hatano†

Abstract

We study a simple open quantum system with a $\mathcal{PT}$-symmetric defect potential as a prototype model to illustrate a number of general features of $\mathcal{PT}$-symmetric open quantum systems. One key feature is the resonance in continuum (RIC), which appears in both the discrete spectrum and the scattering spectrum of such systems. The RIC wave function forms a standing wave extending throughout the spatial extent of the system, and in this sense represents a resonance between the open environment associated with the leads of our model and the central $\mathcal{PT}$-symmetric potential. We also illustrate that as one deforms the system parameters, the RIC may exit the continuum by splitting into a bound state and a virtual bound state at the threshold, a process which should be experimentally observable. We also study the exceptional points appearing in the discrete spectrum at which two eigenvalues coalesce; we categorize these as either EP2As, at which two real-valued solutions coalesce before becoming complex-valued, and EP2Bs, for which the two solutions are complex on either side of the exceptional point. The EP2As are associated with $\mathcal{PT}$-symmetry breaking; we argue that these may be more stable against parameter perturbation than the EP2Bs. We also study complex-valued solutions of the discrete spectrum for which the wave function is nevertheless spatially localized, something that is not allowed in traditional open quantum systems; we illustrate that these may form quasi-bound states in continuum (QBICs) under some circumstances. We also study the scattering properties of the system, including perfect transmission states and a connection with the bound states in the discrete spectrum.

References


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Peculiar form of pseudo-Hermiticity in two-sided deformation of Heisenberg algebra

Alexandre Gavrilik∗

joint with

Ivan Kachurik†

Abstract

Recently we have introduced the two-parameter \((p,q)\)-deformed and three-parameter \((p,q,\mu)\)-deformed extensions of the Heisenberg algebra, and explored [1] these extensions under requirement of their being related with certain (nonstandard) deformed quantum oscillator algebras. In the present work we show that such relatedness leads, instead of usual Hermitian conjugation or familiar \(\eta\)-pseudo-Hermitian conjugation [2], to the extended rules of \(\eta(N)\)-pseudo-Hermitian conjugation [3] of the creation and annihilation operators, with \(\eta(N)\) depending on the particle number operator \(N\). In conjunction with this we also deduce that the position and momentum operators obey particular \(\eta(N)\)-pseudo-Hermiticity [3], while the involved Hamiltonian remains Hermitian. Diverse cases of such \(\eta(N)\)-based conjugation and \(\eta(N)\)-pseudo-Hermiticity are analyzed, and interesting implications considered.

References


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PT-symmetric interpretation of the electromagnetic self-force

Gianfreda Mariagiovanna∗
joint with
Carl M. Bender†

Abstract

It is well known that a charged oscillating particle emits an electromagnetic field, and because the particle is charged it interacts with this field. This phenomenon is described by what is called a self-force [1]. The motion of the particle is expressed by the so-called Abraham-Lorentz equation

$$m \tau \dddot{x}(t) - m \dddot{x}(t) - x(t) = 0,$$

where the characteristic time $\tau$ depends on the particle’s mass $m$ and the charge $e$. Because this equation is third order, its solutions suffer from physical inconsistencies such as runaway modes and pre-acceleration, which imply that the energy is not conserved.

Here the Bateman approach [2] for constructing the Lagrangian for a dissipative system has been extended to include the motion of the radiating particle $x(t)$. Introducing an additional degree of freedom $y(t)$, namely a time reversed companion of $x(t)$, we show that the system composed by $x(t)$ and $y(t)$ is a $PT$-symmetric conservative system that could be derived from a Hamiltonian [3], but the $PT$ symmetry is broken at both classical and quantum level. However, by allowing the charged particles to interact and by adjusting the coupling parameters to put the model into an unbroken $PT$-symmetric region, one eliminates the classical nonrelativistic runaway modes and obtains a corresponding nonrelativistic quantum system that is in equilibrium and ghost free. The quantum version of the coupled system allows an unbroken $PT$-symmetric region that corresponds exactly to the $PT$-symmetric unbroken region in the classical model, the eigenfunctions are normalizable in the complex eight-dimensional phase-space and their spectrum is real.

References


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A new type of PT-symmetric random matrix ensembles

Eva-Maria Graefe*

joint with

Steve Mudute-Ndumbe*, Matthew Taylor*

Abstract

Recently there have been much efforts towards the introduction of new classes of non-Hermitian random matrix models respecting $PT$-symmetry. Here we add to these efforts by proposing two new random matrix ensembles as universality classes for matrices with a real characteristic polynomial, that is, $PT$-symmetric matrices. These are ensembles of Gaussian split-Hermitian and split-self-dual matrices, related to the split signature versions of the complex and the quaternionic numbers. These matrices have either real or complex conjugate eigenvalues, the statistical features of which we investigate in detail for $2 \times 2$ matrices. The advantage of this class of matrices compared to previous suggestions is that it is straightforward to investigate the $N \times N$ case as well, for which we shall discuss various results.

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Analytic calculation of non-adiabatic dynamics around an exceptional point

Naomichi Hatano\(^*\)

joint with

Rikugen Takagi\(^†\)

Abstract

We analytically compute the non-adiabatic dynamics around an exceptional point of a $2 \times 2$ non-Hermitian matrix. By numerically integrating the Schrödinger equation, Gilary, Mailybaev and Moiseyev [1] found a very interesting phenomenon in the non-adiabatic dynamics. It is well-known that the two eigenstates that would coalesce at the exceptional point are swapped into each other if we encircle the exceptional point quasi-statically. Surprisingly, Gilary et al. noticed that if we encircle it non-adiabatically starting from any states, the final state is one of the two eigenstates; if we encircle it in the other direction, the final state is the other one.

We here confirm this observation by using a model dynamics that allows us to compute the non-adiabatic time-evolution analytically; the final result is obtained as analytic functions. In the calculation, we utilized a useful triangularization devised by Hashimoto [2].

References


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PT-symmetric single-mode micro-ring laser

Matthias Heinrich∗
joint with
Hossein Hodaei, Mohammad-Ali Miri,
Demetrios N. Christodoulides, and Mercedeh Khajavikhan

Abstract

We experimentally demonstrate single-mode operation in PT-symmetric microring laser arrangements operating at telecommunication wavelength. This is achieved by appropriately accompanying an active microring resonator with a lossy partner. The single-mode operation is then achieved through selective PT-symmetry-breaking of this system in favor of one specific mode [1]. While all the other modes reside symmetrically in both of the gain and loss cavities, the broken-symmetry mode remains mostly in the gain cavity and therefore enjoys lasing [2]. The demonstrated scheme is versatile and can be applied to a wide class of dielectric laser cavities in order to enforce single mode operation.

References


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Resonances at and around third-order exceptional points

W.D. Heiss*
joint with
G. Wunner†

Abstract

We analyze scattering cross sections at and near third-order exceptional points (EP3), points in physical parameter space where three energies and eigenfunctions coalesce. At an EP3, the Green’s function contains a pole of third order, in addition to poles of second and first order. We show that the interference of the three pole terms produces a rich variety of line shapes at the exceptional point and in its neighbourhood. This is demonstrated by extending previous work to a system of three driven coupled damped oscillators. The similarities and the differences in the behaviour of the corresponding amplitudes of the classical problem and the scattering cross sections in the quantum mechanical problem are discussed at and near the EP3.

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\( \mathcal{PT} \) systems without parity symmetry: topological states and the \( \mathcal{PT} \) phase diagram

Yogesh N. Joglekar *

Abstract

\( \mathcal{PT} \)-symmetric systems are traditionally characterized by a real, parity-symmetric, kinetic Hamiltonian and a non-Hermitian, balanced gain-loss potential. I will present a new class of lattice models in which the tunneling Hamiltonian is not parity-symmetric, and yet the models have a tunable, positive \( \mathcal{PT} \)-breaking threshold in presence of a pair of gain-loss impurities \( \pm i\gamma \) located at reflection-symmetric sites. I will show that a hidden symmetry is instrumental to the finite threshold strength, and discuss its implications for topological edge-states that remain robust in the \( \mathcal{PT} \)-broken phase. These predictions substantially broaden possible realizations of a \( \mathcal{PT} \) system, particularly in optical waveguide arrays or coupled microstructures, by eliminating the parity-symmetry constraint.

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An angle on invisibility

H. F. Jones∗.

Abstract

The $PT$-symmetric optical grating with index profile $e^{i\beta z}$ has been shown[1] to have the interesting property of being essentially invisible for light incident from one side, while possessing greatly enhanced reflection at a particular wavelength for light incident from the other side. We extend our previous analysis of this grating[2] to obtain an analytic solution for the case when the grating is embedded on a substrate, with different refractive indices on either side. We also generalize the previous case of normal incidence to incidence at an arbitrary angle. In that case the enhanced reflection occurs at a particular angle of incidence for a given wavelength. Finally we discuss how the grating may be used to give lasing.

References


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Spontaneous breaking of a PT-symmetry 
in the Liouvillian dynamics 
at a nonhermitian degeneracy point 

Kazuki Kanki* 
joint with 
Kazunari Hashimoto‡, Tomio Petrosky‡, and Satoshi Tanaka∗

Abstract

We have found it a commonplace phenomenon that a pair of eigenvalues of the hermitian Liouville-von Neumann operator (Liouvillian) changes from pure imaginary to complex with a common imaginary part in an extended function space outside the Hilbert space. Such a transition point is an exceptional point where nonhermitian degeneracy occurs and both the pairs of eigenvalues and eigenvectors coalesce. The transition can be attributed to a spontaneous breaking of a kind of PT-symmetry. Here the PT-symmetry means that the effective Liouvillian anti-commutes with an anti-linear operator $PT$, where $P$ is a linear operator representing a symmetry corresponding to parity and $T$ is the complex conjugation. This PT-symmetry is intrinsic in the Liouvillian dynamics, in contrast to the fact that in “PT-symmetric quantum mechanics” PT-symmetry often appears as a result of phenomenological assumptions, such as a complex valued potential energy.

In the kinetic equation for a particle coupled with a bath the flow term drives the system to PT-symmetry breaking as the wave number of the spatial inhomogeneity gets larger. PT-symmetric eigenmodes with purely imaginary eigenvalues correspond to diffusive processes and eigenmodes in a PT-symmetry broken phase lead to damping wave propagation. We illustrate different behaviors with regard to presence or absence of PT-symmetry with the one-dimensional (1D) quantum Lorentz gas [1] and a 1D polaron model [2] as examples.

References


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Metamaterials, complex refraction and an indefinite Laplacian on a rectangle

David Krejčiřík

Abstract

We investigate the nonelliptic differential expression “div sgn grad” on a rectangular domain in the plane. The seemingly simple problem to associate a selfadjoint operator with the differential expression in an $L^2$ setting is solved here. Such indefinite Laplacians arise in mathematical models of metamaterials characterised by negative electric permittivity and/or negative magnetic permeability. This is joint work with Jussi Behrndt.

References

On Scattering Theory Methods in Studies of Non-self-adjoint Schrödinger Operators

Sergii Kužel*

Abstract

One of important problems of PTQM-related studies is the description of quantitative and qualitative changes of spectrum of a non-self-adjoint Schrödinger operator $A = -d^2/dx^2 + V_\varepsilon(x)$ when complex parameters $\varepsilon$ characterizing the non-symmetric potential $V_\varepsilon(x)$ run certain admissible domain. A typical picture is the following:

[I] non-real eigenvalues ↔ [II] spectral singularities, ↔ [III] similarity to a self-adjoint operator

Obviously, an operator $A$ corresponding to [I] cannot be realized as a self-adjoint operator. While, operators from [III] turn out to be self-adjoint with respect to a new inner product which is equivalent to the initial one. The part [II] can be interpreted as a boundary between [I] and [III]. If an operator $A$ corresponds to [II], then its spectrum is real but $A$ cannot be made self-adjoint by an appropriative choice of equivalent inner product. This phenomenon deals with the appearing of ‘wrong’ spectral points of $A$ which are impossible for the spectra of self-adjoint operators. Traditionally, these spectral points are called exceptional points if they belong to the discrete spectrum of $A$ and spectral singularities in the case of the continuous spectrum.

It is naturally to suppose that the change of spectral properties discussed above can be described with the use of scattering theory methods (more precisely, in terms of poles of $S$-matrices of non-self-adjoint Schrödinger operators $A$). We verify this hypothesis for certain classes of non-self-adjoint Schrödinger operators considered in [1, 2, 3].

References


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Non-Hermitian Hamiltonian with $\mathcal{PT}$ Symmetry for a Modulated Jaynes-Cummings Model

Margherita Lattuca$^*$

joint with

Fabio Bagarello$^†$, Roberto Passante$^†$, Lucia Rizzuto$^†$, Salvatore Spagnolo$^†$

Abstract

Following the recent advances of $\mathcal{PT}$-symmetry in quantum optics, we consider a two-level atom interacting with a single cavity mode of the electromagnetic field in the rotating wave approximation, when one parameter of the system is periodically modulated in time. We show that under an appropriated choice of the parameter characterizing the system’s modulation and after a time average, the system’s dynamic can be described by a static non-Hermitian Jaynes-Cummings Hamiltonian with $\mathcal{PT}$-symmetry. Finally, we generalize the diagonalization of this non-Hermitian Hamiltonian in terms of pseudo-bosons and pseudo-fermions.

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Spontaneous breakdown of $PT$ symmetry in exactly, semi-analytically and numerically solvable potentials

Géza Lévai

Abstract

One of the most characteristic features of $PT$-symmetric quantum mechanics is that tuning some parameters may generate spectacular transitions in the energy spectrum of a Hamiltonian. A typical mechanism is that increasing the strength of the imaginary potential component (i.e. non-hermiticity) pairs of real energy eigenvalues merge and pairs of complex conjugate energy eigenvalues appear. However, this phenomenon, which can be interpreted as the spontaneous breakdown of $PT$ symmetry, does not occur in all the $PT$-symmetric potentials, furthermore the sequence through which the energy eigenvalues turn complex with increasing non-hermiticity may differ from case to case.

Since the introduction of $PT$-symmetric quantum mechanics, considerable experience has accumulated concerning exact analytical, semi-analytical and numerical solutions of $PT$-symmetric potentials (see e.g. [1, 2, 3, 4] and references). The energy eigenvalues of these systems are known, and the spontaneous breakdown of $PT$ symmetry has also been described in many cases. Here we present a comprehensive analysis of these potentials and study the transition of the energy eigenvalues from the real to the complex domain. The mathematical structure of the potentials, as well as that of their shape shows remarkable variety, nevertheless, some of their features may have rather similar character.

References


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Indefinite linear pencils

Michael Levitin*

Abstract

We consider the spectrum of a linear operator pencil $P(\lambda) := A - \lambda B$, with self-adjoint operator coefficients $A$ and $B$. If either $A$ or $B$ is sign-definite, then this spectral problem is reduced to that for a self-adjoint operator, and the spectrum is purely real. If however both $A$ and $B$ are sign-indefinite, the spectrum of the pencil may contain non-real eigenvalues. In many applications, these eigenvalues unexpectedly lie on or under some smooth curves in the complex plane (possibly in a particular asymptotic regime). We analyse several such problems starting from a very simple matrix model of [1], in which $A$ is a tri-diagonal Toeplitz matrix depending on a parameter, and $B$ is a diagonal matrix, with diagonal elements $+1$ or $-1$. Even this seemingly trivial setting requires a deep analysis and opens fascinating interplay with such diverse topics as diophantine number theory and orthogonal polynomials. We shall also address the behaviour of related random models and of corresponding pseudospectra.

References


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Floquet Theory for Periodic Pseudo-Hermitian Hamiltonian

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Abstract
Using Floquet decomposition of the evolution operator $U(t) = Z(t)e^{iMt}$ associated with the periodic pseudo-hermitian Hamiltonian $H(t) = H(t + T)$, we extend Berry’s formulation of the geometric phase to the case of non-adiabatic cyclic evolution for a pseudo-hermitian Hamiltonian ([1, 2, 3, 4]). A two-level pseudo-hermitian system is discussed as an illustrative example.

References

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Embedding $\mathcal{PT}$-symmetric BEC subsystems into closed hermitian systems

Jörg Main∗
joint with
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Holger Cartarius∗, Günter Wunner∗

Abstract

In open double-well Bose-Einstein condensate systems which balance in- and outfluxes of atoms and which are effectively described by a non-hermitian $\mathcal{PT}$-symmetric Hamiltonian $\mathcal{PT}$-symmetric states have been shown to exist. We tackle the question of how these in- and outfluxes can be realized and introduce three model systems in which $\mathcal{PT}$-symmetric subsystems are embedded into a closed hermitian system: a four-mode matrix model, a system with $\delta$ potentials, and a system with smooth potentials. We show that in all three cases the subsystems can have $\mathcal{PT}$-symmetric states. In addition we examine what degree of detail is necessary to describe the $\mathcal{PT}$-symmetric properties and the bifurcation structure of such a system correctly. We also investigate which properties the wave functions of a system must fulfill to allow for $\mathcal{PT}$-symmetric states. In particular the role of the phase difference between different parts of the system will be analyzed.

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Singular Amplification in non-Hermitian Optics

Konstantinos G. Makris,*
joint with
Li Ge,† Hakan E. Türeci,‡
Stefan Rotter*

Abstract

The central theme of this talk is that of power amplification in non-hermitian photonic media. These composite structures, that combine gain and loss, have novel functionalities and many applications in photonics. After a short review of the recent advances in the field of parity-time PT-symmetric optics, we are going to focus on transient amplification effects in lossy amplifiers [1]. Such non-hermitian environments are characterized, in many practical cases, by an overall dissipation. The systematic analysis of power amplification, based on the singular values of the corresponding propagator, is going to be presented in realistic systems of coupled waveguides and cavities [2].

References


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†Department of Engineering Science and Physics, CSI-CUNY, New York, USA.
‡Department of Electrical Engineering, Princeton University, Princeton, New Jersey, USA.
\textbf{$\mathcal{PT}$-symmetric perturbations of a Harmonic Oscillator Operator}

Boris Mityagin
(The Ohio State University, Columbus, Ohio, USA)

A perturbed harmonic oscillator

\[ L = -\frac{d^2}{dx^2} + x^2 + w(x) \]  

(*)

gives an illustration of many phenomena when we analyze:

(a) spectra $\Sigma(z)$ of operators

\[ M = A + izB \]

in a Hilbert space $\mathcal{H}$, where $A$ is a self-adjoint operator with discrete spectrum $\{E_n\}$, and $B$ is “subordinated” to $A$;

(b) convergence of spectral decompositions $S_n$ of such operators $M$;

(c) behavior of eigenvalues $E_n(z), E_n(0) = E_n$, in the complex plane or on the real line.

We’ll give a survey of our results (mine or joint with J. Adduci, P. Djakov, P. Siegl, J. Viola) on (a), (b), (c), with focus on the operator (*) with $w(x) = i\gamma s(x)$, $s(x)$ odd and real-valued, and either $s \in L^p(\mathbb{R}), 1 \leq p < \infty$, or $s$ being a multi-delta function.
Dynamical Formulation of Scattering Theory, Unidirectional Invisibility, and Optical Potential Engineering

Ali Mostafazadeh∗

Abstract

We outline a dynamical formulation of time-independent scattering theory in one dimension and discuss some of its conceptual consequences and practical applications. In particular, we use this formulation to devise a general model for unidirectional invisibility. We employ the latter to propose a local inverse scattering scheme which allows us to give an explicit construction of scattering potentials with any desired scattering properties at any prescribed wavelength. Concrete applications of this scheme include the design of threshold lasers, coherent perfect absorbers, unidirectional and bidirectional optical absorbers, amplifiers, phase shifters, and a variety of invisibility cloaks in one dimension.

References


∗Koç University, Turkey
On the pseudospectrum of the harmonic oscillator with imaginary cubic potential

Radek Novák∗

Abstract

We study the Schrödinger operator with a potential given by the sum of the potentials for harmonic oscillator and imaginary cubic oscillator and we focus on its pseudospectral properties. A summary of known results about the operator and its spectrum is provided and the importance of examining its pseudospectrum as well is emphasized. This is achieved by employing scaling techniques and treating the operator using semiclassical methods. The existence of pseudoeigenvalues very far from the spectrum is proven, and as a consequence, the spectrum of the operator is unstable with respect to small perturbations and the operator cannot be similar to a self-adjoint operator via a bounded and boundedly invertible transformation. It is shown that its eigenfunctions form a complete set in the Hilbert space of square-integrable functions; however, they do not form a Schauder basis.

∗Czech Technical University in Prague
Entanglement properties of two-particle quantum dots near autoionization threshold

Anna Okopińska*
joint with
Arkadiusz Kuroś *

Abstract

In recent years, the application of quantum information tools has contributed new insights into the physics of highly correlated many-body states [1]. In particular, the entanglement entropies started to be used to analyze quantum phase transitions with much effort focused on spin systems. New experimental possibilities of fabricating semiconductor quantum dots of various geometry with a small number of constituents and controllable interactions between them gave an impetus for studying theoretically various few-body models subjected to external potentials. Especially interesting is the discussion of their different characteristics in dependence on varying parameters.

Here, we analyse the entanglement properties of two Coulombically interacting particles in various external potentials. We calculate the linear and von Neumann entropies of the lowest states in dependence on the interaction strength between the particles. Since the considered states become autoionizing resonances above the critical value of the interaction strength, we determine their wave-functions from diagonalization of the non-Hermitian Hamiltonian obtained by complex-coordinate rotation. We discuss how the stability properties of the systems are characterized by the entanglement between the particles. In our calculations for highly elongated systems, the critical behavior of entanglement entropies is observed near the ionization threshold. We compare the entanglement characteristics with the ones for spherically symmetric quantum dots obtained in Ref. [2].

References


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Spontaneous breaking of a pseudo-Hermitian ensemble with real eigenvalues

Mauricio P. Pato*
joint with
Gabriel Marinello*

Abstract

In an effort to provide an ensemble of random matrices which could, in principle, model aspects of PT symmetric systems, it has been shown[1] that by breaking the Hermitian condition of the tridiagonal matrices of the β-ensemble[2], an ensemble of non-Hermitian tridiagonal matrices is obtained in which all the eigenvalues are real and have special statistical properties. Still taking the β-ensemble as a starting point, we now have inverted the approach of Ref. [1] by constructing instead an ensemble of pseudo-Hermitian random matrices whose eigenvalues are the same of the β-ensemble. This shows, as a bonus, that pseudo-Hermiticity does not induce any particular statistical spectral property. Considering the other aspect of PT symmetric systems, namely the spontaneous transition from real to complex eigenvalues[3], we show that the matrices of this new pseudo-Hermitian ensemble undergo a spontaneous breaking in which their eigenvalues move in conjugate pairs into the complex plane[4]. The mechanism of this transition is analytically explained. It is also shown that the eigenvectors of the complex eigenvalues show delocalization as compared with those of the real ones. The sensitivity of the matrices is confirmed by a pseudospectra analysis.

References


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Entangled States in Complex Spacetime

Farrin Payandeh*

Abstract

Negative energy states are applied in Krein space quantization approach to achieve a naturally renormalized theory. For example, this theory by taking the full set of Dirac solutions, could be able to remove the propagator Green function’s divergences and automatically without any normal ordering, to vanish the expected value for vacuum state energy. However, since it is a purely mathematical theory, the results are under debate and some efforts are devoted to include more physics in the concept. Whereas Krein quantization is a pure mathematical approach, complex quantum Hamiltonian dynamics is based on strong foundations of Hamilton-Jacobi (H-J) equations and therefore on classical dynamics. Based on complex quantum Hamilton-Jacobi theory, complex spacetime is a natural consequence of including quantum effects in the relativistic mechanics, and is a bridge connecting the causality in special relativity and the non-locality in quantum mechanics, i.e. extending special relativity to the complex domain leads to relativistic quantum mechanics. So that, considering both relativistic and quantum effects, the Klein-Gordon equation could be derived as a special form of the Hamilton-Jacobi equation. Characterizing the complex time involved in an entangled energy state and writing the general form of energy considering quantum potential, two sets of positive and negative energies will be realized. The new states enable us to study the spacetime in a relativistic entangled "space-time" state leading to 12 extra wave functions than the four solutions of Dirac equation for a free particle. Arguing the entanglement of particle and antiparticle leads to a contradiction with experiments. So, in order to correct the results, along with a previous investigation [1], we realize particles and antiparticles as physical entities with positive energy instead of considering antiparticles with negative energy. As an application of modified descriptions for entangled (space-time) states, the original version of EPR paradox can be discussed and the correct answer can be verified based on the strong rooted complex quantum Hamilton-Jacobi theory [2, 3]. Finally, Comparing the two approaches, we can point out to the existence of a connection between quantum Hamiltonian dynamics, standard quantum field theory, and Krein space quantization and as another example we can use the negative energy states, to remove the Klein’s paradox without the need of any further explanations or justifications like backwardly moving electrons. [4, 5].

References


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Pseudo-Hermitian mass extension in an intensive magnetic field

V.N.Rodionov*

Abstract

Modified Dirac-Pauli equations that are entered using $\gamma_5$-mass factorization $m \rightarrow m_1 \pm \gamma_5 m_2$ of ordinary Klein-Gordon operator, are considered. We also consider the interaction of fermions with an intensive uniform magnetic field, focusing on their $(g - 2)$ gyromagnetic factor. Due to effective research procedures pseudo-Hermitian Hamiltonians exact solutions of energy spectra are derived taking into account the spin of the fermions. The basic research methods are the elucidation of the new border areas of unbroken $\mathcal{PT}$ symmetry of Non-Hermitian Hamiltonians. In particular, it is shown that the real energy spectrum can be expressed by limiting the intensity of the magnetic field $H \leq H_{\text{max}} = m^2/(2\Delta \mu m_1)$, where $\Delta \mu$ is anomalous magnetic moment of particles. see[1-4]

References


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Constant-intensity waves
in non-Hermitian potentials

Stefan Rotter∗
joint with
Konstantinos G. Makris,∗,† Ziad Musslimani,‡ Demetrios Christodoulides§

Abstract

In all of the diverse areas of science where waves play an important
role, one of the most fundamental solutions of the corresponding wave
equation is a stationary wave with constant intensity. The most familiar
example is that of a plane wave propagating in free space. In the presence
of any Hermitian potential, a wave’s constant intensity is, however, imme-
diately destroyed due to scattering and diffraction. In my talk, I will show
that this fundamental restriction is conveniently lifted when working with
non-Hermitian potentials. In particular, I will present a whole new class
of waves that have constant intensity in the presence of linear as well as
of nonlinear inhomogeneous media with gain and loss [1]. These solutions
allow us to study, for the first time, the fundamental phenomenon of mod-
ulation instability in an inhomogeneous environment. Our predictions can
be verified by combining recent advances in shaping complex wave fronts
with new techniques to fabricate non-Hermitian scattering structures with
gain and loss.

References

Constant-intensity waves and their modulation instability in non-Hermitian

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Geometrical and Asymptotical Properties of Non-Selfadjoint Induction Equations with the Jump of the Velocity Field. Time Evolution and Spatial Structure of the Magnetic Field.

Andrei I. Shafarevich∗
joint with
Anna I. Allilueva†

Abstract

We study properties of linear and non-linear induction equations, describing magnetic field evolution in highly conducting fluid. This model is quite popular in astrophysics while studying structures of magnetic fields in stars, planets and galaxies and is closely connected with famous dynamo theory. We suppose that the velocity field of the fluid has a jump near a certain surface. We discuss time evolution and spatial structure of highly localized fields in the limit of high conductivity. In the linear approximation we describe the appearance of delta-shock structures and instantaneous growth of the localized magnetic structures. For the non-linear model we study the instantaneous growth of the field as well as the influence of the curvature of the magnetic sheet on its time evolution.

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Bifurcation of nonlinear eigenvalues in problems with an antilinear symmetry

Petr Siegl∗
joint with
Tomáš Dohnal†

Abstract

Many physical systems can be described by nonlinear eigenvalues and bifurcation problems with a linear part that is non-selfadjoint e.g. due to the presence of loss and gain. The balance of these effects is reflected in an antilinear symmetry, like e.g. the $\mathcal{PT}$-symmetry, of the problem. Under this condition we prove in [1] that the nonlinear eigenvalues bifurcating from real linear eigenvalues remain real and the corresponding nonlinear eigenfunctions remain symmetric. The abstract results are applied in a number of physical models (Bose-Einstein condensates, optics, superconductivity) and additional 2D models are studied numerically.

References


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Hidden symmetry from supersymmetry for matrix non-Hermitian Hamiltonians

Andrey V. Sokolov

Abstract

We consider \( n \times n \) matrix linear differential operators of different, in general, orders that intertwine two \( n \times n \) matrix non-Hermitian, in general, Hamiltonians of Schrödinger form. The notions of weak and strong (in)dependence for these operators are introduced. An operation different for \( n \geq 2 \) from transposing and Hermitian conjugation is offered that maps any matrix intertwining operator into a matrix operator which intertwines the same matrix Hamiltonians in the opposite direction. This operation possesses by many properties analogous to ones of transposing and is identical to the latter in the scalar case \( n = 1 \). With the help of the offered operation we construct polynomial algebra of supersymmetry for any matrix intertwining operator and find criterion of weak (in)dependence for two matrix intertwining operators. For the case of two weakly independent intertwining operators a hidden symmetry operator is built from these operators with the help of the operation mentioned above and properties of the hidden symmetry operator are investigated. Some illustrative examples for the constructions described above are presented.
Time-dependent optomechanical systems and non-Hermitian Hamiltonians

Salvatore Spagnolo

joint with

Fabio Bagarello† Margherita Lattuca‡ Roberto Passante*,
Lucia Rizzuto*

Abstract

Recent results have shown the possible physical interest of $\mathcal{PT}$-symmetric non-Hermitian Hamiltonians to describe specific physical systems such as optical waveguide lattices or complex crystals. We will consider optical and optomechanical systems, achievable experimentally, characterized by time-dependent boundary conditions or time-dependent physical parameters, relevant also for the dynamical Casimir and Casimir-Polder effect. We will show that they can be appropriately described by non-Hermitian Hamiltonians with $\mathcal{PT}$-symmetry. We will then briefly introduce a non-hermitian generalization of the well-known Jaynes-Cummings Hamiltonian that describes a two-level atom interacting with a cavity field mode, when the atomic transition frequency or the field mode frequency is periodically modulated in time.

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‡ Dipartimento di Fisica e Chimica, Università degli Studi di Palermo and CNISM
Phase integrals method in the problem of quasiclassical localization of spectrum

S. A. Stepin*

Abstract

An approach based on phase integrals method will be outlined that enables one to examine quasiclassical asymptotics of spectrum for non-selfadjoint singularly perturbed operators. This approach is applied then to boundary eigenvalue problem for second order differential operators with PT-symmetric cubic potentials of generic type. Bohr-Sommerfeld quantization rules are derived to describe the location of the spectrum and geometric properties of the corresponding spectrum concentration curves are investigated as well.

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A note on invariant biorthogonal sets

Salvatore Triolo\textsuperscript{*}.
joint with
Fabio Bagarello\textsuperscript{†}

Abstract

We show how to construct, out of a certain basis, a second biorthogonal set with similar properties. This general procedure works in very different conditions. In particular, we apply the procedure to coherent states. We also comment on a simple application of the construction to pseudo-hermitian quantum mechanics.

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\textsuperscript{†}DEIM, Università degli Studi di Palermo, Italy, and INFN, Torino, Italy
Sampling Type Operators and their Applications to Digital Image Processing

Gianluca Vinti∗

Abstract

We introduce the sampling type operators and we study their properties together with some approximation results in case of bounded, continuous and uniformly continuous functions; moreover we give a modular approximation theorem for functions belonging to Orlicz spaces (see e.g., [2,1,4]).

Finally, in order to obtain some concrete applications to Digital Image Processing, we discuss some algorithms based on the previous theory and we show how they can be useful in several fields (see, [3,5]).


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Stabilizing Non-Hermitian Systems by Periodic Driving

Qing-hai Wang∗

joint with

Jiangbin Gong∗

Abstract

The time evolution of a system with a time-dependent non-Hermitian Hamiltonian is in general unstable with exponential growth or decay. A periodic driving field may stabilize the dynamics because the eigenphases of the associated Floquet operator may become all real. This possibility can emerge for a continuous range of system parameters with subtle domain boundaries. It is further shown that the issue of stability of a driven non-Hermitian Rabi model can be mapped onto the band structure problem of a class of lattice Hamiltonians. As an application, we show how to use the stability of driven non-Hermitian two-level systems to simulate a spectrum analogous to Hofstadter’s butterfly that has played a paradigmatic role in quantum Hall physics.

References


∗National University of Singapore
Group theoretic approach to rationally extended shape invariant potentials

Rajesh Kumar Yadav*,
joint with
Nisha Kumari*, Avinash Khare†, Bhabani Prsad Mandal*

Abstract

The exact bound state spectrum of rationally extended shape invariant real as well as $PT$ symmetric complex potentials are obtained by using potential group approach. The generators of the potential groups are modified by introducing a new operator $U(x, J_3 \pm \frac{1}{2})$ to express the Hamiltonian corresponding to these extended potentials in terms of Casimir operators. Connection between the potential algebra and the shape invariance is elucidated.

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Three Hilbert space representations of quantum systems

Miloslav Znojil*

Abstract
First we recollect that Hamiltonians $H$ (and/or other observables $A$ with real spectra: typically, PT-symmetric ones) which appear non Hermitian in a friendly but false Hilbert space $\mathcal{H}^{(F)}$ of quantum states may be interpreted, in Dyson’s $\Omega$-map spirit, as simplified isospectral partners of the conventional self-adjoint textbook Hamiltonians $h$ (and/or other observables) living in a primary (but, by assumption, prohibitively complicated) physical Hilbert space $\mathcal{H}^{(P)}$. We remind the auditorium that the point of the theory is that an ad hoc change of inner product may transform $\mathcal{H}^{(F)}$ into a much simpler physical Hilbert space $\mathcal{H}^{(S)}$, unitarily equivalent to $\mathcal{H}^{(P)}$. Finally a few applications of the formalism will be discussed covering the topics of relativistic kinematics, discrete and non-local interactions, unitarity of scattering and of a fall into instability at an exceptional-point time.

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PART II

POSTERS
Complex spectrum of the Liouvillian and transport process in 1D quantum Lorentz gas

Kazunari Hashimoto∗

Kazuki Kanki†, Satoshi Tanaka†, and Tomio Petrosky‡

Abstract
We discuss non-equilibrium transport process in a weakly-coupled one-dimensional quantum Lorentz gas based on the complex spectrum of the Liouvillian. We apply the well known Brillouin-Wigner-Feshbach method to the eigenvalue problem of the Liouvillian. The effective Liouvillian thus obtained takes a non-Hermitian form, and it can have complex spectrum. It is known that the effective Liouvillian is identical to the collision operator in non-equilibrium statistical mechanics [1]. Thanks to the simplicity of the system, we have successfully obtained an analytic solution of the eigenvalue problem for arbitrary value of a wavenumber \( k \), which is a measure of spatial inhomogeneity of the particle distribution.

In this talk, we shall focus on the structure of the complex spectrum of the Liouvillian in non-hydrodynamic situation with emphasis on their relation to transport process in the system. There we shall show that the spectrum has various interesting structures. For example, the spectrum has a structure that reflects form of the interaction potential between particles, and it leads to an interesting “beating” process in time evolution of the Wigner distribution function. We shall also show that the spectrum has the exceptional point (EP), which is a branch point singularity in the parameter space at which both eigenvalues and eigenvectors coalesce and thus it leads to the Jordan block, at the point where \( k \) becomes equal to inverse of mean-free-length of a particle. It also leads to the telegraph equation for the Wigner distribution function. At the EP, a PT-symmetry is broken. There we introduce a new representation of a non-Hermitian operator with EPs, which includes a generalized Jordan block form, in order to analyze the spectral structure near EPs [2].

References

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Non-Hermitian Hamiltonians with a real eigenvalue

Boubakeur Khantoul*

Abstract

Using the Lewis-Riesenfeld invariant operator method, we study the time evolution of the time-dependent non-hermitian Hamiltonians and derive the pseudo-hermiticity relation. As an application, we have treated the time-dependent pseudo-hermitian linear harmonic oscillator.

*University of Algeria
Lateral confinement effect on stability of two-electron quantum dots

Arkadiusz Kuroś

Abstract

Semiconductor quantum dots are theoretically described as Schrödinger systems of few Coulombically interacting electrons in an external potential which models their geometry. In open potentials the ionization process may be considered. Discussion of autoionization has been performed for spherically symmetric quantum dots in terms of their size and capacity parameters [1, 2] and the interaction between the electrons [3]. Here, we consider a highly anisotropic two-electron quantum dot with the lateral confinement modelled by a harmonic potential and the longitudinal one by an attractive Gaussian potential which can be approximated as quasi-one-dimensional system with an effective Coulomb interaction

\[ V_{\text{eff}}(x) = \sqrt{\frac{\pi}{l^2}} e^{\frac{x^2}{2l^2}} \left[ 1 - \text{erf} \left( \frac{x}{\sqrt{2}l} \right) \right], \]

where \( l \) is the oscillator length [4].

Apart from bound states, such a system exhibits resonances that are related to the ionization process in the longitudinal direction. The resonance states will be determined using complex-coordinate rotation method [5], which requires non-Hermitian quantum mechanical approach. The detailed analysis of the effect of the lateral and longitudinal confinement on the properties of anisotropic quantum dots is performed. We show how the efficacy of ionization in the longitudinal direction depends on the shape of the lateral trapping potential. In addition, we discuss the strictly one-dimensional limit.

References


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Computational Results in Two Systems of
Pseudo-Hermitian Random Matrices

Gabriel Marinello∗
joint with
Mauricio Porto Pato∗

Abstract

It has been shown [1] that by breaking the Hermitian condition of the
matrices of the β-ensemble [2], an ensemble of non-Hermitian tridiagonal
matrices in which all the eigenvalues are real is obtained. This non-
Hermitian ensemble was constructed as an effort to provide, in the RMT
context, an ensemble whose matrices could, in principle, model aspects
of PT symmetric systems. Changing the approach, we constructed two
pseudo-Hermitian ensembles derived from the general β ensemble. One
is such that its matrices are isospectral with those of the β-ensemble, on
which we also introduced a perturbation term which breaks the pseudo-
Hermiticity. In the other, we introduced sign changes in the off-diagonal
elements which preserve the pseudo-Hermiticity of the matrix. We per-
formed numerical experiments with matrices of these ensembles and present
our findings, particularly their spectral behavior as the eigenvalues move
into the complex plane.

References


(2002).

∗Instituto de Física, Universidade de São Paulo, São Paulo, S.P., Brazil
Here we present a reconstruction method by means of a family of sampling Kantorovich operators with applications to thermographic images, (see e.g., [2, 1, 3, 4]). Approximation properties of these operators have been developed in different settings, as the space of bounded and continuous/uniformly continuous functions and the more general setting of Orlicz spaces, both important for their applications to Signal/Image Processing.

The mathematical theory of these operators shows how it is possible to reconstruct and to enhance multivariate signals, such as images. In particular, we are able to reconstruct images taken from thermographic survey of masonry walls, and to enhance their quality. In recent years thermographic images have been largely used in civil engineering to make diagnosis and monitoring of buildings. Moreover these images are used both to assess actual dimensions of structural elements and to identify the masonry texture, i.e. the mutual arrangement of the blocks (made of stones and/or bricks) and mortar joints inside the wall portion analyzed.

In order to obtain a consistent texture of the masonry we apply digital images algorithms and we will show that the texture obtained by the application of our theory is more realistic from an engineering point of view and more fitting to the real structure and therefore allows us to make an accurate structural analysis of the building.

Finally, the reconstruction methods are used to estimate the elastic characteristics of the masonry walls of a real-world case-study. The mechanical properties allows us to analyze the response of a masonry structure under seismic actions in terms of modal analysis.

In conclusion our model, based on the developed theory of sampling Kantorovich operators, suggests a method to overcome some difficulties that arise when dealing with the vulnerability analysis of existing buildings and allows us to estimate the mechanical characteristics of the masonries using non-destructive tests with consequent advantages in terms of operativeness and costs, (see e.g., [5, 6]).

References

A new type of $PT$-Symmetric random matrix ensembles

Steve Mudute-Ndumbe*

joint with

Eva-Maria Graefe*, Matthew Taylor*

Abstract

Here we construct new matrix ensembles over alternative algebra spaces of split-complex and split-quaternionic numbers. We build ensembles comprised of matrices which are Hermitian with respect to these algebras. These matrices have real characteristic polynomials, and are thus equivalent to $PT$-Symmetric matrix Hamiltonians. We give a brief introduction to split-complex numbers and split-quaternions, elaborating their useful properties; and a brief introduction to Random Matrix Theory, exploring the importance of the field. Finally, we derive explicit results for the spectral densities of our new ensembles in the 2 x 2 case, and numerically identify properties in the more general N x N case.

*Imperial College London.
A Bound on the Pseudospectrum of the Harmonic Oscillator with Imaginary Cubic Potential

Frank Rössler*
joint with
Patrick Dondl, Patrick Dorey*

Abstract
We are concerned with the non-normal operator
\[ H = -\frac{d^2}{dx^2} + ix^3 + cx^2 + bix \]
on \( \text{dom}(H) = \{ \phi \in L^2(\mathbb{R}) \mid H\phi \in L^2(\mathbb{R}) \} \), where \( c > 0, \ b \geq 0 \) are constants. It is well known that this operator is m-accretive and thus generates a one-parameter contraction semigroup \( e^{-tH} \). Furthermore, it was shown by Dorey, Dunning, and Tateo in [1] (see also [2]) that the spectrum of \( H \) is real. The \( \epsilon \)-pseudospectrum of the operator, however, contains an arbitrarily large set for any \( \epsilon > 0 \) ([3, 4]) and thus does not approximate the spectrum in a global sense.

By exploiting the fact that the semigroup \( e^{-tH} \) is compact for \( t > 0 \), we show a complementary result, namely that for every \( \delta > 0, \ m \in \mathbb{N} \) there exists an \( \epsilon > 0 \) such that
\[ \sigma_\epsilon(H) \subset \{ z : \Re(z) \geq \lambda_m - 1 \} \cup \bigcup_{n=0}^{m} \{ z : |z - \lambda_n| < \delta \}, \]
where \( \lambda_n \) denotes the \( n \)-th eigenvalue of \( H \). This proves that the \( \epsilon \)-pseudospectrum of \( H \) converges locally in the Hausdorff metric to the spectrum.

References

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General dynamical description of quasi-adiabatically encircling exceptional points

Stefan Rotter*,
joint with
Thomas J. Milburn†, Jörg Doppler*, Catherine A. Holmes‡, Stefano Portolan†, Peter Rabl†

Abstract
The appearance of so-called exceptional points in the complex spectra of non-Hermitian systems is often associated with phenomena that contradict our physical intuition. One example of particular interest is the state-exchange process predicted for an adiabatic encircling of an exceptional point. In this work [1] we analyze this process for the generic system of two coupled oscillator modes with loss or gain. We identify a characteristic system evolution consisting of periods of quasi-stationarity interrupted by abrupt non-adiabatic transitions. Our findings explain the breakdown of the adiabatic theorem as well as the chiral behavior noticed previously in this context [2, 3] through the switching between two fixed points in the dynamics and the phenomenon of stability loss delay. The framework we set up to describe these effects provides a unified approach to model quasi-adiabatic dynamical effects in non-Hermitian systems in a qualitative and quantitative way.

References

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Classical and Quantum Dynamics in the (non-Hermitian) Swanson Oscillator

Alexander Rush∗
joint with
Eva-Maria Graefe∗,
Hans Jürgen Korsch†
Roman Schubert‡

Abstract
Here I will present an exploration of the classical limit and the dynamics of a popular model PT-symmetric system: the non-Hermitian quadratic oscillator known as the Swanson oscillator. I will give a full classical description of its dynamics using recently developed metriplectic flow equations, which combine the classical symplectic flow for Hermitian systems with a dissipative metric flow from the anti-Hermitian part [1]. Since the Hamiltonian is quadratic, the classical dynamics exactly describe the quantum dynamics of Gaussian wave packets. I will show that the classical metric and trajectories, as well as the quantum wave functions, can diverge in finite time even though the PT-symmetry is unbroken, i.e., the eigenvalues are purely real.

References

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