

Exercise Sheet 3

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Exercise 1. Describe the adjunctions $\Sigma_f \dashv \Delta_f \dashv \Pi_f$ in **Set**.

Exercise 2. Describe the LCCC structure of presheaf categories.

Exercise 3. Let

$$\begin{array}{ccc} B' & \xrightarrow{\beta} & B \\ f' \downarrow & & \downarrow f \\ A' & \xrightarrow{\alpha} & A \end{array}$$

be a pullback square in a locally cartesian closed category. Construct a natural transformation

$$\Sigma_{f'} \Delta_{\beta} \Rightarrow \Delta_{\alpha} \Sigma_f$$

Show that it is a natural isomorphism. Prove an analogous statement for the right adjoints to pullback.

Exercise 4. Given $f : B \rightarrow A$, $g : C \rightarrow B$ in an LCCC, consider the diagram

$$\begin{array}{ccccc} & & \Delta_f \Pi_f(C) & \xrightarrow{f'} & \Pi_f(C) \\ & \swarrow \varepsilon & \downarrow \beta & & \downarrow \alpha \\ C & & B & \xrightarrow{f} & A \\ & \searrow g & & & \end{array}$$

construct a natural isomorphism

$$\Sigma_{\alpha} \Pi_{f'} \Delta_{\varepsilon} \Rightarrow \Pi_f \Sigma_g$$

Explain how this is analogue to the type-theoretic axiom of choice.