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Aspects of Critical Thinking in Classroom Instruction of Secondary School Mathematics Teachers in Jordan

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Abstract: The aim of this study is to investigate the critical thinking aspects appearing in secondary mathematics classrooms in Amman, Jordan, as shown from the teacher's behaviors. Thirty-eight lessons, each for a different teacher, were analyzed according to a model of classification of critical thinking aspects. This model was empirically developed during the study. The results showed that secondary mathematics teachers are not oriented to improving critical thinking among students, as very little of the classroom episodes contained general or specific aspects of critical thinking. The developed model this study provides can be used as a tool to assess mathematics classes regarding improving critical thinking.

Problem

This study has emerged from two basic considerations: (a) The development of critical thinking is a basic aim of education in Jordan (MOE, 1996) and (b) the teaching behavior of teachers is the most important factor that affects the development of critical thinking among students.

Based on these considerations, the study attempted to investigate aspects of critical thinking in the classroom teaching behavior of secondary mathematics teachers, and thus, was specifically directed to answer the following questions:

1. What aspects of critical thinking (abilities and dispositions) are reflected in the teaching behavior of secondary mathematics teachers?
2. To what extent do the aspects of critical thinking dominate the teaching behavior?

Definitions

Critical thinking: that sort of thinking that deals with what one should believe or do in any situation or event. Two main features mark this thinking. The first is that it is reasonable thinking that leads to deductions and sound decisions justified and supported by acceptable proofs. The second is that it is reflective thinking that shows a complete awareness of the thinking steps that lead to the deductions and the decisions.

Dominance of critical thinking aspects: the number of episodes in a single lesson that include an aspect or more of critical thinking, relative to the total episodes of the lesson.

Sample

The population was all the secondary sections (classes) in Amman. The sample for the study consisted of 38 secondary sections. The sections were chosen using the random proportional stratified procedure, in which class level (first secondary, second secondary, and third secondary) and gender were taken as stratifying variables. Thirty-eight teachers were thus selected, one teacher for each section. The sections sample represented 5% of the population and 25% of the secondary mathematics teachers.

Method

To obtain a complete sample of the teaching behavior of the sample teachers, one class session was recorded on tape for each and then transcribed by the author. In order not to be affected, the teachers of the sample did not know about the purpose of the study when they were asked to record their lesson. The taped lessons were from different chapters (geometry, algebra, calculus, statistics, and probability).

Constructing the Classification Model of Critical Thinking Aspects

To answer the research questions, a classification model has been modified to analyze the teaching behavior. This model consists of two main parts; the first deals with the teacher's behaviors (acts) that include general aspects of critical thinking and have nothing to do with the mathematics content. The second part is dealing with the teaching behaviors that reflect an aspect of critical thinking and are related in a direct way to the mathematics content. The first part was divided into abilities and dispositions, in which the teachers' behavioral acts reflect the abilities and the dispositions appearing in Ennis's classification of critical thinking (Ennis, 1985). The second contains four subparts related to the mathematics content: concepts, generalizations, skills, and problem solving.

The following table represents this model. Table 1

Classification Model of Critical Thinking Aspects

1. General aspects
    - 1.1 Abilities
        - 1.1.1 Focusing on a specific issue

1.1.2 Keeping the main point in mind
1.1.3 Asking clarification questions
1.1.4 Asking explanation questions
1.1.5 Taking into consideration students' both right and wrong opinions and discussing them
1.1.6 Connecting previous knowledge with new
1.1.7 Being precise in using statements and symbols
1.1.8 Providing information in a systematic way, highlighting the logical sequence
1.1.9 Stating consistency among statements

1.2 Dispositions
1.2.1 Stressing the necessity of identifying the goal and what should be done before starting answering
1.2.2 Stressing the necessity of identifying the given information before starting answering
1.2.3 Encouraging students to search for necessary information
1.2.4 Encouraging students to examine the solution obtained
1.2.5 Encouraging students to represent the information using tables, graphics, maps, etc.

2. Content-related aspects
2.1 Concepts
2.1.1 Identifying the concept's characteristics
2.1.2 Comparing the concept with other concepts
2.1.3 Identifying examples of the concept with providing justifications
2.1.4 Identifying counterexamples of the concept with providing justifications

2.2 Generalizations
2.2.1 Determining the concepts contained in the generalization and the relations among them
2.2.2 Determining the conditions of applying the generalization
2.2.3 Determining the different formulas of the generalization (special situation)
2.2.4 Providing support evidence for the generalization

2.3 Algorithms and skills
2.3.1 Clarifying the conceptual base of the skill
2.3.2 Comparing student performance with exemplary performance

2.4 Problem Solving
2.4.1 Setting a general form for the target solution
2.4.2 Determining the given information
2.4.3 Determining the relevance and irrelevance of information
2.4.4 Selecting and justifying a strategy to solve the problem
2.4.5 Determining and deducing the subgoals that lead to the goal
2.4.6 Suggesting alternative method to solve the problem
2.4.7 Determining the similarities and differences between the given problem and other problems

To build this model, a logical empirical method has been used, as shown in the following steps:

1. A sample of 6 lessons was randomly selected from the study of 38 recorded lessons. Each one was partitioned into several classroom episodes. An episode was defined as "part of the lesson, complete in its meaning and with a specific objective." Each episode was analyzed to find out all teaching behavioral acts that the episode contained. A teaching act behavior was defined as "a purposeful act on the part of the teacher within the episode." Usually the behavior acts were related together to enable the teacher to achieve the episode's objective. The teacher may provide information, or encourage students to provide information, or ask clarification questions, or give instructions, or discuss the student's opinion, or answer a question, etc.; all these were considered as behaviors acts.
2. In the light of the definition that this study has adopted for critical thinking, and depending on the Ennis taxonomy of critical thinking, each behavior act has been judged according to the following standard: Does this behavior act display any ability or disposition of critical thinking? In other words, does this behavior show that the teacher is directed (with or without awareness) to show critical thinking ability or disposition?

Accordingly, all the behaviors were categorized into two categories; related and not related to critical thinking. The related behaviors were categorized into general acts that do not relate essentially to the mathematics content and specific acts related to the mathematics content.

3. Another 6 lessons were analyzed, in which few new behavioral acts related to critical thinking appeared. Thus it seemed that the deduced list was enough and it is valid to represent the critical thinking

aspects.
The general behavioral acts were separated from the specific acts. The general acts then were separated into abilities and dispositions. After that the behavioral acts in the abilities and dispositions categories were reviewed in order to put similar ones together.
4. The specific behavioral acts were categorized according to the mathematical content, which are concepts, generalizations, skills and algorithms, and problem solving.
5. The deduced taxonomy of this study was compared with Ennis's taxonomy by four external referees and it was found that all the points in the study taxonomy have a parallel point in Ennis's taxonomy, in spite of some difference in labels and details. It is important to mention that Ennis's taxonomy focused on the general logical and evaluating processes related to critical thinking without mentioning any practical situations, whilst the deduced taxonomy has balanced the logical processes with instruction content and the teacher instructional movements.

Analyzing the Classroom Lessons

The following procedures were carried out to determine critical thinking aspects:
1. The lesson was divided into several (from 11-18) episodes.
2. Each episode was labeled according the mathematical content, i.e., what it was about (concepts, generalizations, skills, problem solving).
3. From each episode, all the teacher behavioral acts were listed and categorized according to the deduced taxonomy, as follows:
   - 3.1 Behavioral act contained a general aspect (ability or disposition) of critical thinking.
   - 3.2 Behavioral act contained a specific aspect of critical thinking (related to mathematical content).
   - 3.3 Behavioral act contained an aspect that obstructed the improvement of critical thinking.
   - 3.4 Behavioral act was not related to critical thinking.
4. According to the above work, four types of classroom episodes were identified:
   - 4.1 Positive episodes, containing behavioral acts that reflected at least one aspect of critical thinking and did not contained any obstructive behavior.
   - 4.2 Negative episodes, containing behavioral acts that reflected only a negative or a hindering aspect of critical thinking.
   - 4.3 Null episodes, containing any behavioral act that was evaluated as positive or negative.
   - 4.4 Neutral episodes, containing both positive and negative behavioral acts.

Examples of the Analysis

Example 1

Episode from First Secondary Grade

Lesson: Distance between a Point and a Line

Teacher: *Notice with me now that in order to find our target, which is the cosine of this angle, we have to find the length of "a m". I know "a w"—what is its length?*
Students: *Four*
Teacher: *So "a w" equals 4. If we know "m w"…*
Suha: *It's two.*
Teacher: *How you did know that it is two?*
Suha: *Because it is in the middle.*
Teacher: *Middle! How?*
Suha: ……
Teacher: *Can we know the coordinates of the point m, Suha?*
Suha: *The x coordinate…is the distance from y—the axis that fixed distance.*
Teacher: *How much is this distance?*
Suha: *Three.*
Teacher: *So we know the x coordinate; it is three. Let's come to the y coordinate.*
Students: *Four.*
Teacher: *Do we know the distance between the point and the x-axis?*
Students: *No.*
Teacher: *Let's call her y unknown. I have now a point on a line; one of its coordinates is known and the other is unknown. How can we find the unknown?*
Students: *From the line equation.*
Teacher: *Yes, we set x and solve for y.*

A student solves for y and finds that y = 2.
Teacher: *So y = 2, as Suha guessed.*
Amal: *We can find this distance through congruent triangles.*
Teacher: *How?…Why?*
Amal: *We have two opposite correspondent angles…*
Teacher: *What else?*
Amal*: .....*
Teacher: *The congruent parts in these two triangles are the right angles and those two opposite angles… that's it …*
Amal: *Yes, we cannot say that the two triangles are congruent.*
Teacher: *We said that y coordinate is equal to 2. So the distance "m w" is equal to 2. Accordingly the distance "a w" is …how much?*
Students: *Four.*
Teacher: *Yes, "m w" equals 2 and thus "a m" equals 2.*
The analysis of the above episode is clarified in the following table:
Table 2
Analysis of the Episode of Example 1

Content	Behaviors	Includes	Episode Type	Critical Thinking Aspects
Problem solving	-Identified the information that should be found in order to find the target	-Specific	Positive (General + specific)	General: Abilities: -Asking clarification questions -Asking explanation questions -Taking into consideration students' both right and wrong opinions and discussing them
	-Asked clarification questions	-General		
	-Asked explanation questions	-General		
	-Deduced new information	-Specific		
	-Discussed students' wrong opinions	-General		

Example 2.
Episode from First Secondary Grade
Lesson: Equation of the Circle
Teacher: *We have now the second kind of equation of the circle; that is when the center of the circle does not lie on the origin. We have now the equation with a center m (j , k) and radius r. How we will find the equation? The equation is almost the same as the first one .It is* $(x- j)^2 + (y – k)^2 = r^2$*, where j, k are radiuses. For example, if we want to find the center of a circle* $(x- 2)^2 + (y – 1)^2 = 16$.
Students: The *center is 2, 1.*
Teacher: *And the radius?*
Students: *Root 16*
Teacher: *Which is 4. The radius is 4 and the center is (2 , 1).*
The analysis of the above episode is clarified in the following table:
Table 3 Analysis of the Episode of Example Two

Content	Behaviors	Includes	Episode Type	Critical Thinking Aspects
Generalization	-Provided the equation of the circle (in an imprecise way, not organized, without any evidence)	-Hindering	Negative	-Failing to justify the statement
	-Provided an example for the equation	-Not related to critical thinking		-Using statements and symbols in an imprecise way -Producing information in an unorganized way -Failing to study the issue from all sides and in an integrated way

Reliability

To examine the reliability of lesson analysis, two methods were used. The first is finding consistency with the researcher herself. The second is the consistency between the researcher and others. In the first method 10 lessons were reanalyzed. There was complete match between the two analyses. In the second method, 5 lessons were selected and given to five mathematics teachers to analyze according to the described method. The results were compared with the researcher's analysis. The matches were relatively high (93%).

Analysis and Results

The frequency of occurrence of each of the episode types was calculated.

The total number of classroom episodes analyzed in this study was 521. Table 4 displays the distribution of these episodes on the four different types of episodes which have been found in this study, in addition of the distribution of the positive episodes according to the kind of critical thinking aspect, i.e., general or specific.

It can be noticed that around one-fifth of the episodes did not contain any aspect of critical thinking (positive or negative). Around two-fifths of the total episodes were positive episodes; one-quarter were negative episodes.

Table 4

Distribution of Classroom Episodes

Positive	Negative	Null	Neutral	Total
212 40.7% General 33% (2% dispositions) Specific 22% General *AND* Specific 14%	128 24.6%	105 20.1%	76 14.6%	521 100%

Regarding the positive episodes, it can be noticed from Table 4 that most of these episodes (80%) contained just general aspects of critical thinking. Around a fifth of these episodes contained only specific aspects whilst just one-third of these episodes contained both general and specific aspects of critical thinking. From the above results we can say that the percentage of the total 521episodes that contained general and specific aspects of critical thinking together is just 14%.

Negative or Hindering Aspects of Critical Thinking

Following are the negative aspects of critical thinking found in this study:

1. Not taking students' opinions into consideration.
2. Not justifying statements.
3. Reaching generalizations in spite of insufficient given information.
4. Using statements and symbols in an imprecise way.
5. Producing information in an unorganized way.
6. Not studying issues from all sides and in an integrated way.
7. Stating many of the same sorts of examples and repeating statements without any purpose.
8. Digressing from the main point.

Discussion

This study is concerned with two points. The first is what sort of critical thinking aspects the secondary mathematics teacher shows while teaching. The second is the dominance of these aspects in secondary mathematics classrooms.

Regarding the first point, the aspects that have been found in mathematics lessons have been classified to form a suggested taxonomy model of critical thinking aspects. Even though the taxonomy has been theoretically based on Ennis's taxonomy of critical thinking, it is important to say that it has been derived practically from real behaviors of teachers. The question that can be suggested regarding this model is: to what extent is it possible to use this classification as a model to assess the secondary mathematics teacher's behaviors? This question may lead us to ask the following questions: What other suggested aspects should be there in this model? Why have the teachers not shown these suggested aspects?

Regarding the second point of this study, the results showed that less of half of classroom episodes contained positive aspects of critical thinking. Also, the results showed that a quarter of the total episodes contained negative aspects that hinder critical thinking. Taking into consideration that the goal of improving critical thinking is a main goal of educational reform in Jordan, it seems that the secondary mathematics teachers in Amman do not consider the goal of improving critical thinking in a serious way. Also it can be indicated that those teachers do not have clear and correct perceptions about the appropriate instructional behaviors to improve critical thinking.

One can notice from the results that the "general" aspects of critical thinking compared to "specific" aspects that related to the mathematics content dominated in the episodes by one and half times. This means that the little concern that teachers showed regarding improving critical thinking mostly came unrelated to the content they teach. Assuming that the proper frame to improve critical thinking would encompass both the general and specific aspects of critical thinking, one can say that the percentage of episodes that can be considered as "good" episodes to improve critical thinking is just 14%. Again we can say that it seems that teachers do not consider critical thinking in their teaching in a clear, focused, and balanced way. In general their teaching is not oriented toward teaching critical thinking through teaching content.

This oriented of instructional behavior may be explained by several factors, such as:

1. Teachers do not view critical thinking as the original goal, which should lead to the other goals. In this regard more studies are needed.
2. Teachers do not have enough knowledge about the nature of critical thinking or the strategies to teach content through teaching critical thinking. Also, more studies are needed to investigate this factor.
3. As in Jordan teachers depend completely in their teaching on the national curriculum, this may be one of the factors which affects their teaching as it has been explained in this study. Thus it may be worthwhile to analyze this curriculum to see the aspects of critical thinking there.

Assumptions and Limitations

This study is oriented to point out critical thinking aspects in secondary mathematics teachers' instruction. The results that have been found in this study are limited to the following assumptions.

Teachers' behavior in the classroom was analyzed for a sample of secondary mathematics teachers. The teachers' sample was from the area of Amman. As this sample does not differ in its preparation or experience from the other samples from other areas in Amman, it was assumed that the results of this study can be generalized to all secondary mathematics teachers in Amman.

The analysis was restricted to teachers' verbal talking and not students'. Also, the analysis was on one class period (45 minutes) for each teacher, assuming that the sample of teaching behavior that appeared in one period could represent teaching behavior in all periods in spite of the content's changing. The study assumed that it is possible to analyze teachers' instructional behavior to see the aspects of critical thinking that this behavior reflects.

The results of this study are related with the proposed classification system of critical thinking.

References

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