Modeling impulsive insulin delivery in insulin pump with time delays

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Abstract: We continue our attempt of modeling the open-loop control of glucose level by impulsive insulin injections. Several time delays exist in the system. The delays include the time needed for insulin from injection depot to transport to the interstitial compartment, the time for the slow inhibition of the hepatic glucose production (HGP), and the time for insulin secretion caused by the elevated glucose concentration level from remaining functional pancreatic $\beta$-cells. None of them is negligible. The model proposed in this paper incorporates these time delays. Our analytical studies show that all solutions are permanent, a periodic solution exists, and for the case of type 1 diabetes mellitus (T1DM) the periodic solution is unique and globally asymptotically stable. Numerically it has been shown that moderate time delays in the system are beneficial in lowering blood sugar level rather than harmful. In contrast, these time delays cause that the open-loop control takes longer time to lower glucose concentration level. Our studies also elucidate the noticeable inhibitory effect on HGP by the remaining functional $\beta$-cells. Similar to our previous work, we demonstrated that smaller dose with higher delivery frequency has better effect in continuous subcutaneous insulin injection (CSII) administration. We expect that our findings are helpful for clinical therapies.